

BIHEP-TH-95-27, Oct 1995

CP-Violation For $B \rightarrow X_s l^+ l^-$ Including Long-Distance Effects

Dong-Sheng Du^{a,b}, Mao-Zhi Yang^{b *}^a *CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, China*^b *Institute of High Energy Physics, Chinese Academy of Sciences,**P.O.Box 918(4), Beijing, 100039, P. R. China[†]*

PACS numbers: 13.20.-V, 13.20.He

Abstract

We consider the CP violating effect for $B \rightarrow X_s l^+ l^-$ process, including both short and long distance effects. We obtain the CP asymmetry parameter and present its variation over the dilepton mass.

*E-mail address: Duds@bepc3.ihep.ac.cn; Yangmz@bepc3.ihep.ac.cn

[†]mailing address.

As well known, the flavor changing process $b \rightarrow sl^+l^-$ can serve as an excellent “window” for precisely testing the standard model or for finding new physics beyond it. This process occurs through the one-loop diagrams. There are three types of Feynman diagrams for $b \rightarrow sl^+l^-$ transition, they are electromagnetic (photonic) penguin diagrams, weak (Z^0 boson) penguin diagrams, and box diagrams [1,2]. These diagrams produce the short distance contributions to this process. The short distance contribution to the branching ratio of the inclusive process $B \rightarrow X_sl^+l^-$ is estimated to be about 10^{-5} at large mass of top quark [2,3]. In addition to the short distance contributions, there are long distance contributions to $b \rightarrow sl^+l^-$ through physical intermediate states:

$$b \rightarrow s(u\bar{u}, c\bar{c}) \rightarrow sl^+l^-.$$

The intermediate states can be vector mesons such as ρ , ω , J/ψ , and ψ' . The long distance contribution to the branching ratio of $b \rightarrow sl^+l^-$ is calculated to be as large as 10^{-3} [4,5]. So the long distance effect is not negligible. In this paper, we study the long distance effect in the CP -violation of the inclusive process $B \rightarrow X_sl^+l^-$. Our work is different from previous ones in two aspects. Firstly, in Ref.[1], the authors studied the CP -violation effect of $B \rightarrow X_sl^+l^-$ by considering only photonic penguin diagrams, here, we consider all the three types of the diagrams (electromagnetic, weak, and box diagrams) and include QCD corrections within the leading logarithmic approximation [6]. Secondly, we consider both short and long distance contributions.

The effective Hamiltonian relevant to $b \rightarrow sl^+l^-$ transitions is [3,6,7,8]

$$\begin{aligned} \mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left(\frac{\alpha}{4\pi S_w^2} \right) \sum_i V_i & [A_i \bar{s}\gamma_\mu(1 - \gamma_5)b\bar{l}\gamma^\mu(1 - \gamma_5)l + \\ & + B_i \bar{s}\gamma_\mu(1 - \gamma_5)b\bar{l}\gamma^\mu(1 + \gamma_5)l \\ & - 2im_b S_w^2 F_2^i \bar{s}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)/q^2 b\bar{l}\gamma^\mu\gamma_5 l], \end{aligned} \quad (1)$$

where $V_i = U_{is}^* U_{ib}$ ($i = u, c, t$) is the product of CKM matrix elements. $S_w = \sin\theta_w$, θ_w is the Weinberg angle. $l = e, \mu$. q is the momentum of the lepton pair.

At the scale $\mu \approx M_w$, the coefficients A_t and B_t take the forms:

$$\begin{aligned} A_t &= -2B(x) + 2C(x) - S_w^2[4C(x) + D(x) - 4/9] \\ B_t &= -S_w^2[4C(x) + D(x) - 4/9] \end{aligned} \quad (2)$$

where $x = m_t^2/M_w^2$.

$$\begin{aligned} B(x) &= \frac{1}{4} \left[\frac{-x}{x-1} + \frac{x}{(x-1)^2} \ln x \right] \\ C(x) &= \frac{x}{4} \left[\frac{x/2-3}{x-1} + \frac{3x/2+1}{(x-1)^2} \ln x \right] \\ D(x) &= \left[\frac{-19x^3/36 + 25x^2/36}{(x-1)^3} + \frac{-x^4/6 + 5x^3/3 - 3x^2 + 16x/9 - 4/9}{(x-1)^4} \ln x \right]. \end{aligned} \quad (3)$$

Here, $B(x)$ arises from box diagram, and $C(x)$ from Z^0 penguin diagram, while $D(x)$ is contributed from γ penguin diagram. We can see from eq.(3) that with x coming larger, the contribution from box diagram and γ penguin diagram will decline, while $C(x)$ will become dominant. Then using the renormalization group equation to scale the effective Hamiltonian down to the order of the b quark mass, one obtain

$$\begin{aligned} A_t(x, \xi) &= A_t(x) + \frac{4\pi}{\alpha_s(M_w)} \left\{ -\frac{4}{33}(1 - \xi^{-11/23}) + \frac{8}{87}(1 - \xi^{-29/23}) \right\} S_w^2 \\ B_t(x, \xi) &= B_t(x) + \frac{4\pi}{\alpha_s(M_w)} \left\{ -\frac{4}{33}(1 - \xi^{-11/23}) + \frac{8}{87}(1 - \xi^{-29/23}) \right\} S_w^2 \end{aligned} \quad (4)$$

where $\xi = \frac{\alpha_s(m_b)}{\alpha_s(M_w)} = 1.75$.

Moreover, the coefficient for the magnetic-moment operator is given by

$$\begin{aligned} F_2^t(x, \xi) &= \xi^{-16/23} \left[-\frac{1}{12} \frac{8x^3 + 5x^2 - 7x}{(x-1)^3} + \frac{3x^3/2 - x^2}{(x-1)^4} \ln x \right. \\ &\quad \left. - \frac{116}{135}(\xi^{10/23} - 1) - \frac{58}{189}(\xi^{28/23} - 1) \right]. \end{aligned} \quad (5)$$

In our numerical calculation, we take $m_t = 174 \text{ GeV}$ [9]. Furthermore the non-resonant coefficients A_i , B_i (i=u,c) are represented by

$$A_i = B_i = a_2 S_w^2 g \left(\frac{m_i^2}{m_b^2}, \frac{q^2}{m_b^2} \right), \quad (16)$$

with [7]

$$g(r_i, s) = \begin{cases} \frac{4}{3} \ln r_i - \frac{8}{9} - \frac{4}{3} \frac{4r_i}{s} + \frac{2}{3} \sqrt{1 - \frac{4r_i}{s}} (2 + \frac{4r_i}{s}) (\ln \frac{1 + \sqrt{1 - 4r_i/s}}{1 - \sqrt{1 - 4r_i/s}} + i\pi), & (\frac{4r_i}{s} < 1); \\ \frac{4}{3} \ln r_i - \frac{8}{9} - \frac{4}{3} \frac{4r_i}{s} + \frac{4}{3} \sqrt{\frac{4r_i}{s} - 1} (2 + \frac{4r_i}{s}) \arctan \frac{1}{\sqrt{4r_i/s - 1}}, & (\frac{4r_i}{s} > 1). \end{cases} \quad (7)$$

Here $a_2 = C_- + C_+/3$, is the coupling for the neutral $b\bar{s}q\bar{q}$ ($q = u, c$) four-quark operator.

In addition to the short distance contribution, the inclusive decay $B \rightarrow X_s l^+ l^-$ involves the long distance contributions arising from $u\bar{u}$ and $c\bar{c}$ resonances, such as $\rho(770)$, $\omega(782)$, $J/\psi(3100)$, and $\psi'(3700)$ et.c. The long distance contribution to the coefficients A and B in Eq.(1) can be taken as [4,5,10,11]

$$A_v = B_v = \frac{16\pi^2}{3} \left(\frac{f_v}{M_v} \right)^2 \frac{a_2 S_w^2}{q^2 - M_v^2 + iM_v \Gamma_v} e^{2i\phi_v}. \quad (8)$$

where M_v , and Γ_v are the mass and width of the relevant vector meson ρ , ω , J/ψ , and ψ' , respectively. $e^{2i\phi}$ is the relevant phase between the resonant and non-resonant amplitude. The decay constant f_v is defined as

$$\langle 0 | \bar{c} \gamma_\mu c | V(\epsilon) \rangle = f_v \epsilon_\mu. \quad (9)$$

We can determine f_v through the measured partial width for the decays of the mesons to lepton pairs [12],

$$\Gamma(v \rightarrow l^+ l^-) = \frac{4\pi}{3} \frac{(Q_c \alpha)^2}{M_v^3} f_v^2, \quad (10)$$

with $Q_c = \frac{2}{3}$. For the parameter a_2 , there is the CLEO data $|a_2| = 0.26 \pm 0.03$ [15]. In this work, a_2 should be taken as $a_2 = -(0.26 \pm 0.03)$, and $\phi_v = 0$ or $a_2 = 0.26 \pm 0.03$, $\phi_v = \frac{\pi}{2}$ [11].

The differential decay width of the inclusive process $B \rightarrow X_s l^+ l^-$ over the dilepton mass is given by [4]

$$\frac{d}{dz} \Gamma(B \rightarrow X_s l^+ l^-) = \frac{G_F^2 m_b^5}{192\pi^3} \left[\frac{\alpha}{4\pi S_w^2} \right]^2 F_b(z), \quad (11)$$

where $z = \frac{q^2}{m_b^2}$.

$$\begin{aligned}
F_b(z) = & [|V_i A_i(z)|^2 + |V_i B_i(z)|^2] f_1^b(z) + \\
& + S_w^2 \{V_i^* V_j [A_i(z) + B_i(z)]^* F_2^j + H.C.\} f_{12}^b(z) + \\
& + 2S_w^4 |V_i F_2^i|^2 f_2^b(z),
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
f_1^b(z) &= 2(1-z)(1+z-2z^2) \\
f_{12}^b(z) &= 6(1-z)^2 \\
f_2^b(z) &= 4(1-z)(1/z - \frac{1}{2} - z/2).
\end{aligned} \tag{13}$$

We define the CP -violating asymmetry through the rate difference between B and \bar{B} :

$$\mathcal{A}_{cp} = \frac{\Gamma_{\bar{b}} - \Gamma_b}{\Gamma_{\bar{b}} + \Gamma_b} \tag{14}$$

where Γ_b is obtained by integrating Eq.(11) over the dilepton mass squared z from $z_{min} = (\frac{2m_l}{m_b})^2$ to $z_{max} = (1 - \frac{m_s}{m_b})^2$. The CKM matrix in Eq.(12) can be written in terms of four parametres λ , A , ρ and η in the Wolfenstein parametrization[14]. There have been definite results for λ and A , which are $\lambda = 0.2205 \pm 0.0018$ [15] and $A = 0.80 \pm 0.12$ [16]. But for ρ and η , there are not definite results. So we express the CP-violating parameter for $B \rightarrow X_s e^+ e^-$ in terms of ρ and η ,

$$\mathcal{A}_{cp}^{S+L} = \frac{7.618 \times 10^{-3} \eta}{1.799 + 2.991 \times 10^{-3} \rho + 45.876(1 + 0.0484^2 \eta^2) + 1.4718 \times 10^{-4}(\rho^2 + \eta^2)} \tag{15}$$

for the case of including long distance effects, and

$$\mathcal{A}_{cp}^S = \frac{3.1389 \times 10^{-3} \eta}{1.702 + 3.345 \times 10^{-3} \rho + 3.607 \times 10^{-2}(1 + 0.0484^2 \eta^2) + 1.4373 \times 10^{-4}(\rho^2 + \eta^2)} \tag{16}$$

for the case without long distance effects. Eq.(15) and Eq.(16) indicate that η affects the CP asymmetry mainly, and ρ does not.

In table I, we give the results of \mathcal{A}_{cp} for some “best values” of (ρ, η) [16]. We can see that, i) without the long distance effects, the CP-violating asymmetry \mathcal{A}_{cp} is about

$(1.8 \sim 6.1) \times 10^{-4}$, while, in Ref.[1], the relevant CP asymmetry is about 1.3×10^{-2} . Our result is about twenty times smaller than theirs. The reason is that, in Ref.[1], only the photonic penguin is considered. But in fact the Z^0 penguin will give big contribution to the amplitude at large $m_t(\sim 174GeV)$ [2], at the same time, it doesn't provide large CP-nonconserving phase, ii) including the long distance effects, the result of the CP asymmetry parameter \mathcal{A}_{cp} is about $(1.5 \sim 5.4) \times 10^{-5}$. It is reduced about one order by the resonant effects. The main difference between the cases with and without long distance effect resides in the third term of the denominator of eq.(15) and (16), which comes from the integration of the first term of eq.(12), i.e., $\int_{z_{min}}^{z_{max}} dz[|V_c|^2|(A_c(z) + B_c(z))^S|^2 f_1^b(z)]$. Without resonant contributions

$$\int_{z_{min}}^{z_{max}} dz[|V_c|^2|(A_c(z) + B_c(z))^S|^2 f_1^b(z)] = 0.12|V_c|^2, \quad (17)$$

While with resonant contributions

$$\int_{z_{min}}^{z_{max}} dz[|V_c|^2|(A_c(z) + B_c(z))^{S+L}|^2 f_1^b(z)] = 152.9|V_c|^2, \quad (18)$$

Because the total decay width of J/ψ or ψ' is narrow ($\Gamma_{J/\psi} = 88KeV$, $\Gamma_{\psi'} = 277KeV$), when the dilepton mass squared z is near the mass squared of J/ψ or ψ' , the resonance will give a big contribution. At the same time, the first term of eq.(12) only contribute to the decay width Γ_b and $\Gamma_{\bar{b}}$, it does not give contribution to the CP-violation. So with the resonant effects, the CP-violation will be reduced greatly.

We also calculated the distribution of the CP asymmetry over the dilepton mass for (ρ, η) taking the “preferred value” of $(-0.12, 0.34)$ [16],

$$a_{cp} = \frac{F_{\bar{b}}(z) - F_b(z)}{F_{\bar{b}}(z) + F_b(z)}. \quad (19)$$

The result is plotted in Fig.1. The solide line is for the case without resonances and the dotted line for the case with resonances. We can see that, in general, the CP asymmetry

is suppressed by the resonance effect, and in the region near the resonances, CP-violating parameter is suppressed severely.

E. Golowich and S. Pakvasa have discussed the long range effects in $B \rightarrow K^*\gamma$ [17], which is relevant to the condition of the squared mass $q^2 = 0$. They found a small effect by respecting gauge invariance. It should be noted that there is no controversy between their results and ours. That is in Fig.1, it is shown that when $q^2 \rightarrow 0$ the long distance effect is also very small in the case of $B \rightarrow X_s l^+ l^-$.

Finally, we want to point out that for the case of $l = \mu$, the CP asymmetry parameter is smaller than the $l = e$ case.

This work is supported in part by the China National Natural Science Foundation and the Grant of State Commission of Science and Technology of China.

References

- [1] N.G.Deshpande, G.Eilam, and A.Soni, Phys.Rev.Lett.57 (1986) 1106.
- [2] W.S.Hou, R.S.Willey, and A.Soni, Phys.Rev.Lett.58 (1987) 1608.
- [3] N.G.Deshpande and J.Trampetic, Phys.Rev.Lett. 60 (1988) 2583.
- [4] N. G. Deshpande, J. Trampetic, and K.Panose, Phys.Rev.D39 (1989) 1461.
- [5] C.S.Lim, T.Morozumi, and A.I.Sanda, Phys.Lett.B23 (1989) 343.
- [6] B.Grinstein, M.J.Savage, and M.Wise, Nucl.Phys.B319 (1989) 271.
- [7] T.Inami and C.S.Lim, Prog.Theor.Phys. 65 (1981) 297.
- [8] D.S.Liu, Preprint UTAS-PHYS-95-08.
- [9] F.Abe, et al., CDF Collaboration, Phys.Rev. D50 (1994) 2966.
- [10] C.A.Deominguez, N.Paver and Riazuddin, Z.Phys.C48 (1990) 55.
- [11] P.J.O'Donnell and H.K.K.Tung, Phys.Rev.D43 (1991) 2067.
- [12] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. D50 (1994) 1173.
- [13] CLEO Collab., M.S.Alam et al., Phys.Rev.D50 (1994)43.
- [14] L.Wolfenstein, Phys.Rev.Lett. 51 (1983) 1945.
- [15] K.Hikasa et al.(PDG), Phys. Rev. D45 (1992) 1.
- [16] A.Ali and D.London Preprint CERN-TH 7408/94.
- [17] E.Golowich and S.Pakvasa, Phys. Rev. D51 (1995) 1215

(ρ, η)	\mathcal{A}_{cp}^S	\mathcal{A}_{cp}^{S+L}
$(-0.48, 0.10)$	1.81×10^{-4}	1.60×10^{-5}
$(-0.44, 0.12)$	2.17×10^{-4}	1.92×10^{-5}
$(-0.40, 0.15)$	2.71×10^{-4}	2.40×10^{-5}
$(-0.36, 0.18)$	3.25×10^{-4}	2.88×10^{-5}
$(-0.32, 0.21)$	3.79×10^{-4}	3.35×10^{-5}
$(-0.28, 0.24)$	4.34×10^{-4}	3.83×10^{-5}
$(-0.23, 0.27)$	4.88×10^{-4}	4.31×10^{-5}
$(-0.17, 0.29)$	5.24×10^{-4}	4.63×10^{-5}
$(-0.11, 0.32)$	5.78×10^{-4}	5.11×10^{-5}
$(-0.04, 0.33)$	5.96×10^{-4}	5.27×10^{-5}
$(+0.03, 0.33)$	5.96×10^{-4}	5.27×10^{-5}
$(-0.12, 0.34)$	6.14×10^{-4}	5.43×10^{-5}

Table I. CP asymmetries for some “best values” of (ρ, η) . \mathcal{A}_{cp}^S denotes the cases without long distance contributions, \mathcal{A}_{cp}^{S+L} with long distance contributions.

Figure Captions

Fig. 1. The dilepton mass distribution of the CP asymmetry parameter for $B \rightarrow X_s e^+ e^-$ process without resonances (solide line) and with resonances (dotted line).